PROBLEMS FOR SOLUTION.

SEND ALL COMMUNICATIONS ABOUT PROBLEMS TO B. F. FINKEL, Springfield, Mo.

ALGEBRA.

479. Proposed by S. A. COREY, Albia, Iowa.

Prove or disprove

$$\left\{\begin{vmatrix} x & -v & -z \\ -y & -z & v \\ -z & y & -x \end{vmatrix}^2 + \begin{vmatrix} y & -v & -z \\ x & -z & v \\ v & y & -x \end{vmatrix}^2 + \begin{vmatrix} x & y & -z \\ -y & x & v \\ -z & v & -x \end{vmatrix}^2 + \begin{vmatrix} x & -v & y \\ -y & -z & x \\ -z & y & v \end{vmatrix}^2\right\} \div \begin{vmatrix} x & -y & -z & v \\ y & x & -v & -z \\ z & v & x & y \\ v & -z & y & -x \end{vmatrix}^2 = (x^2+y^2+z^2+v^2)^{-1}.$$

480. Proposed by FRANK IRWIN, University of California.

Solve the equation

$$(x-1) - 2\left(1-\frac{1}{x}\right) - 3\left(1-\frac{1}{x}\right)\left(1-\frac{2}{x}\right) - 4\left(1-\frac{1}{x}\right)\left(1-\frac{2}{x}\right)\left(1-\frac{3}{x}\right) - \cdots$$
$$- n\left(1-\frac{1}{x}\right)\left(1-\frac{2}{x}\right)\cdots\left(1-\frac{n-1}{x}\right) = 0.$$

Also the equation

$$(x-a_1) - a_2\left(1-\frac{a_1}{x}\right) - a_3\left(1-\frac{a_1}{x}\right)\left(1-\frac{a_2}{x}\right) - \cdots$$
$$- a_n\left(1-\frac{a_1}{x}\right)\left(1-\frac{a_2}{x}\right)\cdots\left(1-\frac{a_{n-1}}{x}\right) = 0.$$

[Adapted from a formula of Tait's.]

GEOMETRY.

512. Proposed by J. L. RILEY, Northeastern State Normal School, Tahlequah, Okla.

Determine, geometrically, where the circle of curvature at any point of an ellipse again meets the ellipse.

513. Proposed by ALBERT A. BENNETT, University of Texas.

The following construction for angle-trisection was given some years ago in a non-mathematical journal. Let ABC be a right triangle with AB as hypothenuse. Let BD be a ray drawn parallel to AC and extending in the same direction. Let AEF be a variable ray meeting the segment BC in E, and the ray BD in F. Show, by elementary methods, that when the variable ray is so adjusted that $EF = 2AD$, then $\angle EAC = \frac{1}{3} \angle BAC$.

CALCULUS.

427. Proposed by ROGER S. JOHNSON, Adelbert College, Cleveland, Ohio.

Of all ellipses circumscribed about a given parallelogram, the maximum, with regard to area, has as conjugate diameters the diagonals of the parallelogram.

428. Proposed by J. L. RILEY, Northeastern State Normal School, Tahlequah, Okla.

A loop of a lemniscate rolls in contact with the axis of x. Prove that the locus of the node is given by the equation

$$1 + \frac{dy}{dx} = \left(\frac{a}{y}\right)^{\frac{4}{3}}$$

and that $2\rho\rho' = a^2$, if ρ, ρ' be corresponding radii of curvature of this locus and the lemniscate.